**Prediction of Springback in Air-Bending of Advanced High Strength Steel – DP 780**

Xi Yang, Changhyok Choi¹, Nimet Kardes Sever and Taylan Altan*

*Center for Precision Forming (CPF, www.cpforming.edu), The Ohio State University, 339 Baker Systems, 1971 Neil Avenue, Columbus, OH, 43210, USA

*Corresponding author: Phone: 614-292-5063, Fax: 614-292-7219, e-mail: altan.1@osu.edu

**Abstract**

Advanced High-Strength Steels (AHSS) are increasingly used in industry. Thus, the springback prediction in bending of AHSS is important to maintain close geometric tolerances in deformed parts. The unique properties of AHSS: a non-constant E-modulus and a stress-strain relation which does not follow a simple flow stress equation make the prediction of springback very difficult. In this study, an analytical model was developed to predict the springback in air-bending of AHSS, considering the special properties of these materials. The computer code BEND which is used to predict the springback in air-bending of low carbon steels was updated based on the new model. The finite element simulation which includes the properties of the AHSS is also presented in the paper. The comparison between the experimental results and predictions indicates that the detailed consideration of the properties of AHSS affects the accuracy of the springback prediction with the analytical method. It is hoped that this new analysis can be incorporated into the controller of a press brake to adjust the machine for compensating springback during bending.

**Keywords:** V-die bending, Springback, AHSS, Analytical method, Finite element method

1 **Introduction**

Springback is a common phenomenon in sheet metal forming. It refers to the dimensional change of the deformed part after unloading due to the elastic recovery. The prediction of springback in bending of Advanced High-Strength Steels (AHSS) is becoming increasingly important because these materials are now widely used in industry. Springback can be estimated by Finite Element Analysis (FEA). Selections of the appropriate element, solver type, definition of the tool model and boundary conditions are crucial to obtain an accurate solution in reasonable amount of computation time. Furthermore, it is very important to input into the FE model the correct material data such as hardening model, hardening law and E-modulus.
FEA requires a commercial code and computer time. On the other hand, a simple computer code, which can predict the springback fast and accurately, can be incorporated into a press brake controller. Thus, the bending operation can be improved to compensate for springback. Certain properties of AHSS make the springback prediction difficult in using both FEM and analytical methods. First, E-modulus of AHSS is not constant during the forming process. Previous researchers have reported that the unloading and loading elastic moduli of AHSS are not the same and both of them decrease with strain [1-4]. Similar results were also observed in tensile tests conducted at the Center for Precision Forming (CPF) [5, 6]. Most of the methods, used to predict springback, assume a constant E-modulus during deformation. To predict springback accurately, modulus variation has to be considered. Considerable research has been conducted for estimating springback in bending using analytical techniques. Raghupathi et al. [7] proposed an analytical model to predict springback in plane-strain sheet bending. Wang et al. [8] improved this model to develop practical design aids and adaptive control of a press brake. CPF developed a program BEND based on this model which can give an accurate springback prediction in bending of low carbon steels [9]. Two important assumptions were made in this model:

1) Swift’s material model \[\bar{\sigma} = K(\bar{\epsilon} + \bar{\bar{\epsilon}})^n\] was used to describe the sheet metal’s property.
2) The E-modulus was assumed to be constant during the loading and unloading process.

However, AHSS does not fully satisfy these two assumptions. Constant strength coefficient, \(K\), and strain hardening exponent, \(n\), values are not sufficient to describe the material’s behavior in plastic state. Thus, a new analytical model is developed that considers a suitable description of material’s property (flow stress) as well as E-modulus variation. The computer software BEND, which is now used in CPF to predict springback in bending low carbon steels, is now updated based on the new method. In addition, the FEM simulations that consider the unique properties of AHSS are also presented as comparison.

2 Analytical Model
2.1 Material Properties

The unique properties of an AHSS (DP780) are clearly shown in Figures 1 and 2. Figure 1 shows that the unloading modulus, which is obtained from a load – unload tensile test, decreases nonlinearly as the strain increases. Figure 2 illustrates that stress-strain relation, expressed by Swift’s model with constant \(K\) and \(n\), gives a flow stress curve which deviates from the experimental data. These two features of DP780 cause the existing analytical method to fail in predicting springback.
One effective way to improve the methodology for predicting springback in bending of AHSS is to use the E-modulus, as well as $K$ and $n$ values that vary with strain. Thus, the existing equations must be modified. Also, the experimental tabular data from tensile test or bulge test are required to represent $K$, $n$ and E-modulus values. The following steps are necessary:

**Figure 1 Variation of the unloading apparent modulus of DP780 [6]**

**Figure 2 Flow stress data from measurement (bulge test and tensile test, [6]) vs. recreated stress-strain relation using constant K and n values (Material: DP780)**
1) Tabular data of E-modulus vs. strain will be obtained from load – unload tensile test, and flow stress curve will be obtained from the bulge test [6].

2) The flow stress curve will be divided into many small regions according to the recorded strain data points.

3) E-modulus will be interpolated and assumed constant within each small region; different $K$ and $n$ values will be calculated for each small region.

4) New two sets of tabular data will be obtained: E-modulus vs. strain, and $K$ and $n$ vs. strain.

5) Instead of integrating over the entire domain with one set of constant parameters, the maximum moment, elastic stiffness and springback will be calculated by integrating over each small region with different $K$, $n$ and E-modulus values, in function of strain. The final result is obtained by summing up all the integrals in each region.

2.2 Loading Moment in the Plastic Zone

The loading moment in the plastic zone, $M_L$, needs to be determined first. The strain values along the thickness change from “0” at the unstretched fiber to maximum at the inner and outer fibers (Figure 3). Due to the properties of the AHSS, the stress-strain relation will follow different $K$ and $n$ values as strain changes, and E-modulus changes as well. So the thickness of the sheet metal was divided into several small regions, such that the strain values at the boundaries of each region correspond to the experimental data points. Then, within each region, the stress-strain relation can be expressed by Swift’s model with a pair of constant $K$ and $n$. E-modulus can be assumed to be constant within each region since the change of strain is small enough (Figure 3).
Figure 3 Dividing thickness into small regions with respect to tabular strain data and using constant $K$, $n$ and $E$ values corresponding to strain values at different regions.

As a result, the loading moment in the plastic zone ($M_L$) can be calculated by using Eq. (1):

$$
M_L = w \left( \int_{0}^{r_1} \sigma_y dy + \int_{y_1}^{y_2} \sigma_y dy + \cdots + \int_{y_{m-1}}^{r_{out}} \sigma_y dy \right) + w \left( \int_{0}^{y_1} \sigma_y dy + \int_{y_1}^{y_2} \sigma_y dy + \cdots + \int_{y_{m-1}}^{y_m} \sigma_y dy \right)
$$

where $y$ is coordinate at the boundary of each region, $\sigma_y$ is circumferential stress within each region, $r_{in}$ and $r_{out}$ are radii at the inner and outer fibers, respectively (Figure 3).

By using the flow stress relation, Eq. (1) can be modified to Eq. (2):

$$
M_L = w \left( \int_{0}^{y_1} F K_1 (\varepsilon_0 + F \varepsilon_x)^{n_1} dy + \int_{y_1}^{y_2} F K_2 (\varepsilon_0 + F \varepsilon_x)^{n_2} dy + \cdots + \int_{y_{m-1}}^{r_{out}} F K_{n-1} (\varepsilon_0 + F \varepsilon_x)^{n_{m-1}} dy \right) + w \left( \int_{0}^{y_1} F K_1 (\varepsilon_0 + F \varepsilon_x)^{n_1} dy + \int_{y_1}^{y_2} F K_2 (\varepsilon_0 + F \varepsilon_x)^{n_2} dy + \cdots + \int_{y_{m-1}}^{r_{in}} F K_{n-1} (\varepsilon_0 + F \varepsilon_x)^{n_{m-1}} dy \right)
$$

where $F$ is the index to account for anisotropy, $F = [2(1 + \tilde{R})]^{\frac{1}{\bar{M}}}/2[1 + (1 + 2\tilde{R})^{\frac{1}{1-\bar{M}}}(\bar{M}-1)/\bar{M}]$, $\bar{M} = (1+\tilde{R})$ is the anisotropy index, $\varepsilon_x$ is the circumferential strain at each region, $\varepsilon_0$ is pre-strain, $K_1, \ldots, K_n$, $n_1, \ldots, n_m$ are the strength coefficients and strain hardening exponents in different regions. Once the
loading moment in plastic region, $M_L$, is obtained, the moment distribution along the length of the bent sheet, $M(S)$, can be written as a linear function of arc length $S$, as discussed in [8,9].

### 2.3 Stiffness Variation of the Bent Sheet

For low carbon steels, since $E$-modulus is constant, the stiffness of the bent sheet can be calculated as $E'I$ and remains constant through the thickness, where $E'$ is the young’s modulus in plain strain condition ($E' = E/(1 - v^2)$). Based on the classic elastic bending theory, the moment could be expressed in equation (3):

$$M = \left(\frac{1}{r} - \frac{1}{r'}\right)E'I$$  \hspace{1cm} (3)

Where $r$ and $r'$ are the radius of the curvature of the sheet metal before and after the springback.

However, the stiffness of the bent sheet has to be re-calculated to include the effect of $E$-modulus variation for AHSS. We follow the procedure described in section 2.2 to calculate the unloading moment in the plastic region, $M_U$, as given in Eq. (4):

$$M_U = w\left(\frac{1}{r} - \frac{1}{r'}\right) \left(\int_{0}^{Y_1} E'_1 y^2 dy + \int_{Y_1}^{Y_2} E'_2 y^2 dy + \cdots + \int_{Y_m}^{r_{out}} E'_m y^2 dy + \int_{0}^{Y_1} E'_1 y^2 dy + \int_{Y_1}^{Y_2} E'_2 y^2 dy + \cdots + \int_{Y_k}^{r_{in}} E'_k y^2 dy \right) = \left(\frac{1}{r} - \frac{1}{r'}\right) (E'I)_p$$  \hspace{1cm} (4)

where $(E'I)_p$ is stiffness of the sheet in the plastic zone. In the bending operation, the loading moment is equal to the unloading moment ($M_L = M_U$). Thus, the springback can be computed using Eq. (4) if $(E'I)_p$ is calculated first. A more detailed explanation of the moment calculation can be found in [9].

Another issue to consider is that the stiffness along the length of the bent sheet is not constant. Figure 4 illustrates that the strain at outer and inner fibers of the sheet decreases from maximum at plastic zone to zero at the die corner, where at point E which is the boundary of elastic zone, the strain reaches the elastic limit. Since there is only elastic deformation in the elastic zone (E-B), the stiffness in this portion can be calculated by using $E'I$, where $E'$ is the maximum E-modulus. So the equation to calculate springback in the elastic region remains the same:

$$\theta_e = \int_{E}^{B} \frac{M(S)}{E'I} dS$$  \hspace{1cm} (5)

where $\theta_e$ is springback in the elastic region, $M(S)$ is the moment distribution in elastic zone [8,9]. In the elasto-plastic zone, the strains at inner and outer fibers along the length of the bent sheet are different.
As a result, the stiffness at different cross sections is not constant any more. To solve this problem, the arc length is divided into several small regions (1, 2, 3, …, n), as seen in Figure 4. The stiffness of each small section is assumed to be constant and the springback of one region can be calculated by using equation \( \theta_S^n = \int \frac{M}{(E'I)_n} \, dS \), where \((E'I)_n\) is the stiffness of the given section.

Figure 4 Stiffness variation along the length of the bent sheet. In elasto-plastic zone, the arc is divided into small regions and the stiffness in each region is assumed to be constant.

The total springback angle \( \theta_S \) for the bent sheet is obtained by summing up the springbacks of all the small regions in the elasto-plastic zone and also in the elastic zone as given in Eq. (6):

\[
\theta_S = \int \frac{M(S)}{E'I} \, dS = \int_{(1)}^{(2)} \frac{M(S)}{(E'I)_1} \, dS + \int_{(2)}^{(3)} \frac{M(S)}{(E'I)_2} \, dS + \cdots + \int_{(n)}^{(n+1)} \frac{M(S)}{(E'I)_n} \, dS + \left[ \int_{E}^{B} \frac{M(S)}{(E'I)} \, dS \right]
\]

3 Results Obtained from the Mathematical Model

The computer code BEND was modified according to the improved mathematical model to test the effectiveness of the new method. Experimental springback measurements of AHSS DP780 were compared with the predictions made with the program BEND. The experimental results of DP780 are obtained from V-bending tests conducted at Cincinnati Inc. Since during the V-bending test, the sheet metal did not touch the die bottom, the results can be used to compare with air bending test. The dimensions and properties of the tested samples assuming constant \( K \) and \( n \) values are listed in Table 1. Table 2 is the list of the tool dimensions used in V-bending test and BEND program. The die opening is
calculated by the geometric relations between V-bending and air-bending, as shown in Figure 5. The modulus variation from tensile test and flow stress data from bulge test were input for calculation. The evaluation of the program was conducted as follows:

(1) Input the highest and the lowest E-moduli from tensile test while keeping $K$ an $n$ constant. The result from this case can show the influence of E-modulus in springback prediction; (2) Use the E-modulus value in function of the strain in the plastic zone for a given bending angle; (3) consider the E-modulus variation and non-constant $K$ and $n$ in moment and stiffness calculation.

![Figure 5 V-die opening (V) and the equivalent air-bending opening (D_L). $D_L = V - 2R\left[1 - \tan\left(\frac{\pi - \alpha}{4}\right)\right]$, where $\alpha$ is the V-die opening angle ($\alpha=75^\circ$), $R$ is the die corner radius](image)

**Table 1 Properties and dimensions of DP780 sheet samples**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strain hardening exponent ($n$)</td>
<td>0.14</td>
</tr>
<tr>
<td>Strength coefficient ($K$)</td>
<td>1465 MPa</td>
</tr>
<tr>
<td>Initial yield stress ($\sigma_0$)</td>
<td>540 MPa</td>
</tr>
<tr>
<td>Poisson’s ratio ($\nu$)</td>
<td>0.3</td>
</tr>
<tr>
<td>Initial thickness ($t_0$)</td>
<td>1 mm</td>
</tr>
<tr>
<td>Sheet width ($w$)</td>
<td>60 mm</td>
</tr>
<tr>
<td>Friction coefficient ($\mu$)</td>
<td>0.12</td>
</tr>
<tr>
<td>Anisotropy ($\bar{R}$)</td>
<td>0.896</td>
</tr>
</tbody>
</table>

**Table 2 Dimensions used in V-bending test and Air-bending test**

<table>
<thead>
<tr>
<th>Test Type</th>
<th>Punch radius, $R_p$ (mm)</th>
<th>Die corner radius, $R$ (mm)</th>
<th>Die opening (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V-Bending</td>
<td>4.75</td>
<td>3.81</td>
<td>36.77</td>
</tr>
<tr>
<td>Air-Bending</td>
<td>4.75</td>
<td>3.81</td>
<td>32.91</td>
</tr>
</tbody>
</table>
3.1 Springback Prediction with E-modulus in Function of the Largest Strain

To explore the influence of E-modulus variation, first the highest E-modulus (207GPa) when strain is zero and the lowest E-modulus (153GPa) when strain is around 0.1 are used as input to the program, Figure 1. The results are shown in Figure 6. When punch stroke is small (e.g. less than 2 mm), the program did not calculate the springback. This is because when calculating the springback, we assumed there is a pure plastic region near the punch tip which does not contribute to springback. However when the punch stroke is small, the elastic recovery of this region can not be ignored. Therefore the springback estimation is not accurate. It is observed that the springback angles when punch strokes are larger than 3 mm are located between the boundary values predicted using highest and lowest E-moduli. Therefore, the average unloading E-modulus vs. strain (Figure 1) was used as the input to the program next, while $K$ and $n$ are kept constant. For a given bending angle, we first calculate the maximum strains along the length of the bent sheet at inner fiber $\varepsilon_{in}$ and outer fiber $\varepsilon_{out}$. The largest strain is estimated by taking the average of these two values, $\varepsilon_{avg} = \frac{1}{2}(\varepsilon_{in} + \varepsilon_{out})$. E-modulus used for this angle is interpolated from the input E-modulus vs. strain data based on this largest strain value. The predicted results with variable E-modulus are also shown in Figure 6. The results show that the predicted springback with E-modulus as a function of strain is very close to the average measured springback angles.
Figure 6 Predicted springback using E-modulus value corresponding to the largest average strain in bending

3.2 Springback Prediction with Variable $K$, $n$ and E-modulus

In this case, the springback calculation will consider variable $K$, $n$ and E-modulus values in function of strain. Both E-modulus vs. strain and flow stress data obtained from the biaxial VPB test [10], (Figure 7) are input to the BEND program. The loading moment in plastic zone was calculated first using equation (6). As discussed previously, the moment distribution in elasto-plastic and elastic zones can be written as a linear function of the arc length $S$. However instead of using Eq. (5) to calculate springback, the stiffness in elasto-plastic zone was also assumed to be distributed linearly as a function of arc length, $E'I(S)$, Figure 8. Since the stiffness in elastic zone is constant, the springback angle ($\theta_S$) of the whole bent sheet can be calculated as given in Eq. (7):

$$\theta_S = \int_A^B \frac{M(S)}{E'I(S)} dS + \int_E^B \frac{M(S)}{E'I} dS$$  \hspace{1cm} (7)

The predicted results are also plotted in Figure 6. The plot indicates that, using variable $K$, $n$ and $E$ values, the springback predictions are slightly smaller than the calculations using $E$ as a function of the largest strain. This is because if the E-modulus is the function of the largest strain, it is the smallest $E$ value within the part (Figure 1). As a result, the springback calculated using this $E$ value will be larger. While the variable $E$ method considers strain variation within the part, the $E$ value is slight larger than the previous method. Therefore, the estimated springback is smaller. The involvement of variable $K$ and $n$ in this calculation does not show significant influence on the results. The reason is that although the Swift’s flow stress model can not describe the material’s property of AHSS precisely, the deviation between the mathematical model and the experimental data is not that large (Figure 2). Therefore in this case, variable $K$ and $n$ did not considerably improve the estimation accuracy. However, the springback prediction may be improved for different sheet materials and tooling setups.
Figure 7 Flow stress obtained by tensile and viscous pressure bulge (VPB) tests [10]. Data from viscous pressure bulge (VPB) test starts from 0.01 true strain and goes up to 0.35 true strain. Yield strength was found from tensile test and the VPB test data (solid line) was extrapolated for the regions where there is no experimental data (dashed line).
Figure 8 Stiffness variation $E'/I$ along the length of the bent sheet. AE: elasto-plastic zone; EB: elastic zone (Figure 4).

4 Prediction of Results from Finite Element Simulations

Finite element method is another common way to predict springback. Especially in cases where sheet, tool geometry and loading conditions are difficult to model analytically. FEM can give results that are much more accurate if used carefully. Depending on bending geometry, 2D or 3D simulations have to be considered. In most practical bending operations, however, when no flanging around a curved line is involved, 2D simulations give reasonably accurate results. Finite element simulation for the V-die bending test discussed above is presented here as a comparison with analytical method.

The simulation parameters that were used for FE analyses are given in Table 3. A half model was utilized for all FE simulations to save computation time. The flow stress data (Figure 7) is used in tabular form in FEM. In V-bending there is no forward and reverse loading. Thus, isotropic hardening model (Von-Mises) was used in the FE simulations. Bauschinger effect and anisotropy was neglected. Unloading modulus was considered constant in simulations with DEFORM 2D. In addition, a user subroutine was utilized for DEFORM 2D simulations to input the unloading modulus as a variable (Figure 1). The experimental data points were entered in tabular form into DEFORM 2D. Comparison of springback angles obtained from DEFORM 2D and experiments is given in Figure 9.

Table 3 FE simulation parameters for AHSS (DP 780)

<table>
<thead>
<tr>
<th>Simulation parameters</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material type</td>
<td>Sheet - elastic plastic, V-die and punch - rigid</td>
</tr>
<tr>
<td>Element Type</td>
<td>Solid (Brick – DEFORM 2D),</td>
</tr>
<tr>
<td>E-modulus</td>
<td>Constant: 207 GPa and 153 GPa (max. and min. values in Fig. 1)</td>
</tr>
<tr>
<td></td>
<td>Variable (See Fig. 4)</td>
</tr>
</tbody>
</table>
5 Summary and Conclusions

1) The analytical model to predict the springback in bending of low carbon steel has been well developed and proved effective. This method requires that the material has constant E-modulus and its property can be described by Swift’s model \(\bar{\sigma} = K(\epsilon_0 + \dot{\epsilon})^n\) with constant \(K\) and \(n\) values.

2) The challenges in predicting the springback in bending of AHSS are: (1) E-modulus is not constant in bending; (2) Swift’s model is not able to describe accurately the material property of AHSS.

3) An improved analytical model was proposed. Instead of using constant \(K\), \(n\) and \(E\), the strain distribution was divided into small regions. Different E-modulus, \(K\) and \(n\) values, obtained from the experimental data, are assigned for each small region.
4) The equations used to calculate the moment and stiffness in the existing code – BEND – were modified: The moment and stiffness in plastic zone were computed by summing up the integrals over the small divided regions along the thickness.

5) In addition, the stiffness along the length of the bent sheet is not constant in the elasto-plastic zone. The total springback is the sum of the springback values in elasto-plastic and elastic zones.

6) To verify the capability of the program BEND, the springback predictions, based on the new model, were compared with the experimental measurements. The multi-step verification study indicates that the E-modulus has a significant influence on the springback estimation. The accuracy of the estimation can be considerably improved when using E-modulus as a function of the largest strain. When including all the $K$, $n$ and $E$ variations in the computation, the predicted springback angles are slightly smaller than using $E$ as a function of the largest strain. This method does not improve the estimation significantly in this case because the Swift’s model is not accurate but still very close to the experimental data. However considering different sheet materials and thickness, the variable $K$, $n$ and $E$ may help to improve the estimation accuracy.

7) Compared to the analytical method, FE simulations that were performed by using DEFORM 2D with maximum value of unloading apparent modulus showed better agreement with experiments than the predictions with minimum value of unloading apparent modulus obtained using DEFORM 2D. When the variation of unloading apparent modulus was considered in FE simulations by using DEFORM 2D, no improvement in predictions was observed. However, considering variable unloading apparent modulus in FE simulations may improve the springback predictions for different sheet materials and bending angles.

8) Using the flow stress data obtained from tensile test and VPB test gave similar results in FE simulations since the max. effective strain ($\sim 0.07$) in the bending area with the existing tool geometry was smaller than the maximum strain ($\sim 0.12$) obtained from tensile test.

6 Acknowledgements
We would like to thank Edison Welding Institute for conducting load-unload tensile tests to obtain unloading apparent modulus variation and Cincinnati Inc. for allowing us to run V-die bending tests in their facility. We would like to thank Mr. Gautham Sukumaran for his help in the experiments.
References


